

## Propositions and their Converses

Ganita Manjari | Grade 9 | Part II | Chapter 9 | Exercise Set 9.1 — Solutions

For each proposition below: the converse is framed, both statements are judged true or false, true statements are justified, and a counterexample is given for every false statement — following the method shown in the chapter's worked examples.

**Q1. If two lines are parallel, then the corresponding angles formed by a transversal are equal.**

*Converse:*

If the corresponding angles formed by a transversal are equal, then the two lines are parallel.

**Answer:** Both the proposition and its converse are **TRUE**.

**Proposition:** This is a basic result of parallel lines cut by a transversal — corresponding angles are always equal when the lines are parallel.

**Converse:** This is the standard test used to check whether two lines are parallel — if a transversal cuts two lines such that a pair of corresponding angles is equal, the lines must be parallel. (This is the converse of the corresponding-angles axiom, itself taken as an axiom / proved using the properties of angles on a straight line.)

**Q2. If a quadrilateral is a square, then all its angles are equal.**

*Converse:*

If all the angles of a quadrilateral are equal, then it is a square.

**Answer:** Proposition is **TRUE**; Converse is **FALSE**.

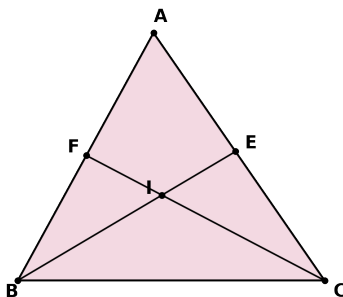
**Proposition:** Every square has all four angles equal to  $90^\circ$ , so this is true by definition of a square.

**Counterexample for converse:** A rectangle that is not a square (e.g. a  $6\text{ cm} \times 3\text{ cm}$  rectangle) has all angles equal (each  $90^\circ$ ), but it is not a square since its sides are not all equal.

**Q3. Given any triangle ABC, the angle bisectors from B and C meet at the incentre I.**

**These bisectors are extended beyond I to meet the opposite sides at E and F respectively.**

**Proposition:** If  $AB = AC$ , then  $IE = IF$ .



*Converse:*

If  $IE = IF$ , then  $AB = AC$ .

**Answer:** Both the proposition and its converse are **TRUE**.

**Proposition ( $AB = AC$  implies  $IE = IF$ ):** If  $AB = AC$ , triangle ABC is isosceles, so angle B = angle C. Since BE and CF are bisectors of angle B and angle C, angle IBC = angle ICB (each half of equal angles). So triangle IBC is isosceles with  $IB = IC$ . Using this together with the equal base angles angle

$B = \text{angle } C$ , triangle IBF and triangle ICE can be shown congruent (ASA:  $\text{angle } IBF = \text{angle } ICE$ ,  $IB = IC$ ,  $\text{angle } BIF = \text{angle } CIE$  as vertically opposite angles), giving  $IF = IE$ .

**Converse ( $IE = IF$  implies  $AB = AC$ ):** By the symmetric argument, if  $IE = IF$ , then triangle IBF is congruent to triangle ICE can be traced back to show  $\text{angle } B = \text{angle } C$ , which makes triangle ABC isosceles with  $AB = AC$ .

**Q4. If  $x = y$ , then  $a + x = a + y$ , where  $x, y$  and  $a$  are any three numbers.**

*Converse:*

If  $a + x = a + y$ , then  $x = y$ .

**Answer:** Both the proposition and its converse are **TRUE**.

**Proposition:** Adding the same number  $a$  to two equal numbers  $x$  and  $y$  keeps them equal — this is the addition property of equality.

**Converse:** If  $a + x = a + y$ , subtracting  $a$  from both sides gives  $x = y$  — this is the cancellation (subtraction) property of equality. Both are routinely used while solving equations, exactly as the chapter states.

**Q5. If  $a$  and  $b$  are perfect squares, then  $ab$  is a perfect square.**

*Converse:*

If  $ab$  is a perfect square, then  $a$  and  $b$  are perfect squares.

**Answer:** Proposition is **TRUE**; Converse is **FALSE**.

**Proposition:** If  $a = m^2$  and  $b = n^2$  for integers  $m, n$ , then  $ab = m^2n^2 = (mn)^2$ , which is a perfect square.

**Counterexample for converse:** Take  $a = 2$  and  $b = 8$ . Neither is a perfect square, but  $ab = 16 = 4^2$ , which is a perfect square.

**Q6. ( $x, y$  are real numbers) If  $x = y$ , then  $x^2 = y^2$ .**

*Converse:*

If  $x^2 = y^2$ , then  $x = y$ .

**Answer:** Proposition is **TRUE**; Converse is **FALSE**.

**Proposition:** If  $x = y$ , multiplying both sides by  $x$  (or  $y$ ) gives  $x^2 = y^2$ . True for all real  $x, y$ .

**Counterexample for converse:** Take  $x = 3, y = -3$ . Then  $x^2 = 9 = y^2$ , but  $x \neq y$ .

**Q7. ( $x, y$  are real numbers) If  $x = y$ , then  $x^3 = y^3$ .**

*Converse:*

If  $x^3 = y^3$ , then  $x = y$ .

**Answer:** Both the proposition and its converse are **TRUE**.

**Proposition:** If  $x = y$ , then multiplying  $x = y$  by  $x^2$  (or  $y^2$ ) repeatedly gives  $x^3 = y^3$ .

**Converse:** Unlike squaring, cubing is one-to-one over the real numbers (a negative number cubed stays negative, a positive number cubed stays positive, and cube root is unique for reals). So  $x^3 = y^3$  forces  $x = y$  — there is no real counterexample here as there was for squares.

**Q8. ( $n$  is a positive integer) If  $n$  is divisible by 24, then it is divisible by both 4 and 6.**

*Converse:*

If  $n$  is divisible by both 4 and 6, then it is divisible by 24.

**Answer:** Proposition is **TRUE**; Converse is **FALSE**.

**Proposition:** Since  $24 = 4 \times 6$ , any multiple of 24 is automatically a multiple of both 4 and 6.

**Counterexample for converse:**  $n = 12$  is divisible by both 4 and 6, but 12 is not divisible by 24. (This happens because 4 and 6 share the common factor 2, so their LCM is 12, not 24.)

**Q9. ( $n$  is a positive integer) If  $n$  is divisible by 60, then it is divisible by both 5 and 12.**

*Converse:*

If  $n$  is divisible by both 5 and 12, then it is divisible by 60.

**Answer:** Both the proposition and its converse are **TRUE**.

**Proposition:** Since  $60 = 5 \times 12$ , any multiple of 60 is a multiple of both 5 and 12.

**Converse:** 5 and 12 share no common factor other than 1 (they are co-prime), so  $\text{LCM}(5, 12) = 60$ . Hence any number divisible by both 5 and 12 must be divisible by their LCM, 60.

**Q10. ( $n$  is a positive integer) If  $n$  is the square of a prime number, then it has exactly 3 factors.**

*Converse:*

If  $n$  has exactly 3 factors, then it is the square of a prime number.

**Answer:** Both the proposition and its converse are **TRUE**.

**Proposition:** If  $n = p^2$  for a prime  $p$ , its only factors are 1,  $p$ , and  $p^2$  — exactly 3 factors.

**Converse:** If  $n$  has exactly 3 factors, pair each factor with its 'partner' (as in the chapter's discussion). Since 3 is odd, one factor-partner pair must repeat, so  $n$  is a perfect square,  $n = f^2$ . The 3 factors of  $n$  must then be 1,  $f$ , and  $f^2$ , which forces  $f$  to be prime (if  $f$  had a smaller factor  $d > 1$ , then  $d$  would be an extra factor of  $n$ , giving more than 3 factors). So  $n$  is the square of a prime.

**Q11. ( $n$  is a positive integer) If  $n$  is a product of two unequal prime numbers, then it has exactly 4 divisors.**

*Converse:*

If  $n$  has exactly 4 divisors, then it is a product of two unequal prime numbers.

**Answer:** Proposition is **TRUE**; Converse is **FALSE**.

**Proposition:** If  $n = p \times q$  with  $p, q$  distinct primes, its divisors are exactly 1,  $p$ ,  $q$ , and  $pq$  — 4 divisors in total.

**Counterexample for converse:**  $n = 8 = 2^3$  has exactly 4 divisors (1, 2, 4, 8), but 8 is not a product of two unequal primes — it is a power of a single prime.

**Q12. ( $n$  is a positive integer) If  $n$  and  $n + 3$  have no factors in common, then  $n$  is not a multiple of 3.**

*Converse:*

If  $n$  is not a multiple of 3, then  $n$  and  $n + 3$  have no factors in common.

**Proposition:** This is easiest to see via the contrapositive: if  $n$  were a multiple of 3, say  $n = 3k$ , then  $n + 3 = 3(k + 1)$  would also be a multiple of 3 — so 3 itself would be a common factor of  $n$  and  $n + 3$ . Since we are given  $n$  and  $n + 3$  share no common factor,  $n$  cannot be a multiple of 3.

**Checking the converse:** Since  $n$  and  $n + 3$  always differ by exactly 3, any common factor of  $n$  and  $n + 3$  must also divide their difference, 3. So the only number that could possibly be a common factor of  $n$  and  $n + 3$  is 3 itself. This means: whenever  $n$  is *not* a multiple of 3,  $n$  and  $n + 3$  cannot share the factor 3 either — so they share no common factor at all.

**Answer:** The converse is also **TRUE** (no counterexample exists). In fact, the proposition and its converse are equivalent here — this is a case, like Example 3 in the chapter, where 'P' and 'Q' imply each other.

**Q13. There are no known 'neat' expressions that generate only primes! Find counterexamples to the following claims.**

(i) All numbers of the form  $4n^2 + 1$  are prime.

**Answer:**  $n = 3$ :  $4(3)^2 + 1 = 4(9) + 1 = 37$  (prime) — try  $n = 4$ :  $4(16) + 1 = 65 = 5 \times 13$ , which is **not prime**. So  $n = 4$  is a counterexample.

(ii) All numbers of the form  $n^2 + n + 11$  are prime.

**Answer:**  $n = 10$ :  $10^2 + 10 + 11 = 100 + 10 + 11 = 121 = 11^2$ , which is **not prime**. So  $n = 10$  is a counterexample. ( $n = 1$  to  $9$  all happen to give primes, which is why this expression can look convincing at first.)

(iii) All numbers of the form  $4^n + 3$  are prime.

**Answer:**  $n = 2$ :  $4^2 + 3 = 16 + 3 = 19$  (prime) — try  $n = 3$ :  $4^3 + 3 = 64 + 3 = 67$  (prime) — try  $n = 5$ :  $4^5 + 3 = 1024 + 3 = 1027 = 13 \times 79$ , which is **not prime**. So  $n = 5$  is a counterexample.

**Q14. Find counterexamples to the following statements.**

(i) If  $n$  is a prime number, then  $2^n - 1$  is a prime number.

**Answer:**  $n = 11$  is prime, but  $2^{11} - 1 = 2047 = 23 \times 89$ , which is **not prime**. So  $n = 11$  is a counterexample. (Numbers of the form  $2^n - 1$  are called Mersenne numbers — not all of them are prime even when  $n$  is prime.)

(ii) If  $n$  is an even number, then  $2^n + 1$  is a prime number.

**Answer:**  $n = 4$  is even, but  $2^4 + 1 = 17$ , which is actually prime — try  $n = 6$ :  $2^6 + 1 = 65 = 5 \times 13$ , which is **not prime**. So  $n = 6$  is a counterexample.

**Q15. Consider the statement: 'If a number is divisible by 8, then it is divisible by both 2 and 4.'**

(i) **Justify the statement:**

**Answer:** Since  $8 = 2 \times 4$ , any number divisible by 8 must also be divisible by both 2 and 4 (just as in Q8, where  $24 = 4 \times 6$ ).

(ii) Is it enough to check divisibility by 2 and 4 to conclude divisibility by 8?

**Answer: No.** This is really asking whether the converse is true, and it is not. Counterexample:  $n = 12$  is divisible by both 2 and 4, but 12 is not divisible by 8. This happens because 2 and 4 are not co-prime (4 is itself a multiple of 2), so their LCM is only 4, not 8 — checking 2 and 4 separately does not give any information beyond just checking 4 alone.

**Q16. Recall the shortcut: a number is divisible by 3 exactly when the sum of its digits is a multiple of 3. Express the relationship between 'a number is divisible by 3' and 'sum of the digits is a multiple of 3' using 'If-then' sentences.**

**Answer:** If a number is divisible by 3, **then** the sum of its digits is a multiple of 3.

If the sum of a number's digits is a multiple of 3, **then** the number is divisible by 3.

Both of these If-then statements (the proposition and its converse) are **TRUE** — that is exactly why this divisibility shortcut works reliably in both directions.

**Q17. Category Q of quadrilaterals: two thin sticks are joined at their midpoints crosswise (as diagonals) to form quadrilateral of type Q. (See Fig. 9.2 in the textbook.)**

(i) **Property:** If a quadrilateral is of type Q, then it has equal-length diagonals.

**Answer: (a)** Yes — the two sticks must be of **equal length**. Since the sticks themselves become the diagonals of the quadrilateral formed, if the property (equal diagonals) is to hold for every quadrilateral of type Q, the two sticks used to build it must already be equal in length.

**(b)** No — it will **not** matter how the two equal sticks are put together (i.e. at what angle or where

they cross); as long as both sticks are equal in length, the resulting quadrilateral will always have equal-length diagonals, because the diagonals simply are the two sticks.

**(ii) Property (converse-style):** If a quadrilateral has equal diagonals, then it is of type Q.

**Answer: (a)** No — the sticks do **not** need to be of equal length for this to make sense as a defining property; rather, type Q is now being *defined* as 'having equal diagonals', so any quadrilateral built from two equal sticks automatically qualifies as type Q by this definition. (Sticks of unequal length would simply produce a quadrilateral that is not of type Q.)

**(b)** No — again the way the two (equal) sticks are crossed does not matter; any quadrilateral formed by two equal-length sticks used as diagonals will have equal diagonals and will therefore be of type Q by definition.

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*Teaching note: Q1, Q2, Q13 and Q14 make good board discussion starters; Q12 is a nice one to let students attempt a counterexample before revealing that the converse is, in fact, also true. Q17 is starred (\*) in the textbook as a more challenging, open-ended question — best used for classroom discussion or as a bonus problem rather than for scored assessment.*